Hydrological scenarios modelling for climate change conditions using the Fokker-Planck-Kolmogorov equation

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Abstract Stochastic-decision-making-procedures (SDMP) are widely used to optimize water resources' administration. SDMP require *a posteriori* probabilistic-density-curve (PDC) (the PDC that characterize the hydrological process for the time frame where the decisions will influence the managed process) as one of the essential inputs to find optimal operational rules to handle a water management device. Stationary stochastic hydrological models (based on stationary Markov chains and ARIMA models) always provide *a priori* PDC (i.e. the PDC that characterize the past (observed time) of a hydrological process), allowing the set-up of a SDMP that increases the investment's security, but reduces the profitability. This paper will show the application of the Fokker-Planck-Kolmogorov (FPK) equation that (in conjunction with a deterministic kernel) forecasts *a posteriori* PDC enabling the SDMP to supply operation rules with a well balanced–cost/benefits relationship. This research is supported by IDEAM (Colombian Institute of Hydrology, Meteorology and Environmental Studies), RSHU (Russian State Hydrometeorological University) and partially by WMO (World Meteorological Organization): one of the aims is to simulate *a posteriori* PDC for hydrological scenarios that could take place in the case of a global warming. Another goal is to provide the above forecast (monthly) for the Colombian Hydropower System. To meet the nationwide domain it was necessary to include ungauged basins in the modelling. The parameterization mechanism for ungauged-basins is presented here.

Key words climate change; probability density function; stochastic decision modelling

INTRODUCTION

The present economic restrictions that have affected all hydrometeorological services have forced the formulation of a new management approach to facilitate a competitive generation of hydrometeorological products. To meet this aim hydrometeorological services are complementing socially oriented services with customer oriented services and customer policies. In this modernization process it is understood that for any water users (customers), the business consists of formulation of the Operation Rules' for water management devices. Doing business as usual, the customer uses SDMP (stochastic-decision-making-procedures) to optimize operation rules application. The customer that applies SDMP has a very high input requirement; first of all he wants to get *a posteriori* PDC (probabilistic-density-curve) required by SDMP and does not care about the data source. Customers do not care about hydrological data collection and the problems of prediction in ungauged basins (PUB): they can not understand where the hydrological network has adequate density of monitoring nodes, or where not, they want SDMP requirements to be satisfied.

Contemporary administration theories about economic value-added (Amat, 2002) and the modern concept of hydrometeorological services (NOAA, 1999) determine that those services have to be competitive to survive in the present neo-liberal market conditions; in light of this guidance, the services have developed the so-called "end to end process for hydrometeorological products building and release". This chain starts with the identification of potential customers and the hydrometeorological products they need. There are two kinds of clients for hydrometeorological services: governmental and economic sectors. For the governmental sector the hydrometeorological products have to provide advanced, short-term forecasts and warnings for the protection of life and property and long-term forecast and scenarios building for water resource management. The second kind of users integrate a wide range of very specific requirements including short- through long-range forecast information for SDMP. Both kinds of users expect customized hydrometeorological products to be included in their processes of management. As presented above, they do not want to be concerned with how those products were built (those worries are essentially for hydrometeorologists): (1) they want to understand how the data (even in ungauged basins) is set up; (2) how hydrometeorological information interacts to develop hydrometeorological scenarios; and (3) how those inputs are offered. Finally, a modernized hydrometeorological service has to care about: what, how and when the users' requirements are to be satisfied. In this way it was established that the main source of uncertainty for users' management systems comes from hydrometeorological products: hydrometeorological services understood the users need of *a posteriori* PDC forecasts to be included in SDMP (both for short and long-term time frames).

Governmental users require long-term hydrometeorological forecasts to formulate guidelines for nationwide level development policies. Actually, the global warming hypothesis is one of the most important key rules in long-term planning. According to IPCC (Intergovernmental Panel for Climate Change) guidelines, hydrometeorological services of countries that integrate the UNFCCC (United Nations Framework Convention on Climate Changes) have to design hydrological scenarios to enable an optimization process for productive sectors' adaptation—facing a climate change. Usually hydrometeorological evaluations are presented in a statistical framework which means that not only the behaviour of mean (expected) values is analysed; but also, the performance of variance and asymmetry of hydrometeorological parameters is studied (Domínguez, 2001). Widely-used methodologies for hydrometeorological scenarios building are unable to produce hydrometeorological scenarios in terms of the three first statistical moments: mean or expected value, variance and asymmetry; but, a way to describe a PDC consists of using those moments (Rozhdiestvienskiy & Chievatoriov, 1974; Kovalenko, 1993). Even the uncertainty of hydrometeorological products in short-term decision making procedures (in the economics sectors) has to be described in the above stated way. In summary, fully deterministic hydrometeorological products are not enough for hydrometeorological services' clients: customers require enhanced probabilistic hydrometeorological products.

STOCHASTIC HYDROLOGICAL MODELLING—FPK EQUATION

There are two widely used approaches for hydrological analyses: deterministic and stochastic. The deterministic approach applies a static water balance, physically based modelling and dynamically conceptual modelling techniques, to offer users the products they require (Feenstra *et al.*, 1998). It is important to remark that up to date (or state-of-the-art) deterministic methodologies are unable to characterize (or forecast) either *a posteriori* or *a priori* PDC (nevertheless, RSHU researchers have been innovating with PDC forecast using just deterministic models and the partially infinite modelling approach, e.g. as Kovalenko (2002) suggested in his work about nonlinear aspects of partially infinite modelling in evolutionary hydrometeoecology). State-of-the-art in the stochastic approach shows us a powerful set of techniques based on the concept of structural function (Gandin, 1963) that is used in random function theory (Kozakievich, 1989). Those techniques make use of ergodicity and stationary properties of random processes. This fact leads to the forecast of *a priori* PDC as a consequence of structural function parameterization through historical data sets. In hydrology the adepts of each approach (deterministic or stochastic) are used to emphasizing a break line between both currents, which means a researcher that usually deals with deterministic models does not believe in random theory. In contrast, appliers of random theory reject the cause–effect principle (this metaphysical opposition is due to the isolated development of both lines in western hydrology). Nevertheless, the development of the theories: (1) stochastic processes and (2) turbulence theory (Frish, 1998), never supposed this divorce between deterministic and stochastic approaches. The stochastic generalization of the process uses a dialectic denial of Laplacian determinism (Kovalenko, 1993), allowing the application of a deterministic kernel and the introduction of uncertainty (noises) to formulate a general process, which behaves as a Markov process and can be described by a one-dimensional conditional PDC). In this sense a process $\xi(t)$ is a Markov process if for any ordered n-moments of time $t_1 < \dots < t \dots < t_n$; n-dimensional conditional probability density depends only on the last fixed value:

$$P(x_n, t_n \mid x_1, t_1; \dots; x_{n-1}, t_{n-1}) = P(x_n, t_n \mid x_{n-1}, t_{n-1}) \tag{1}$$

DETERMINISTIC KERNEL

The following equations examine how a traditional deterministic hydrological model becomes a model that can be described using equation (1). According to Kovalenko (1993) and Dominguez (2002), starting from a two-dimensional (2-D) Saint Venant equation system for non steady flow in the form:

$$\begin{aligned} &\frac{\partial h}{\partial t} + \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} - X = 0 \\ &\frac{\partial h}{\partial x} + Sf_x - So_x = 0 \\ &\frac{\partial h}{\partial y} + Sf_y - So_y = 0 \end{aligned} \tag{2}$$

where: t, time; x, y: spatial coordinates; h, depth of water sheet; $q(x, y)$, water flow; $X(x, y, t)$, precipitation on time unit; Sf_x, friction slope in x–direction; Sf_y, friction slope in y–direction; So_x, terrain slope in x direction; So_y, terrain slope in y direction.

Through spatial integration we come to the following agglutinated model as presented by Dominguez (2002):

$$\tau(t,Y,X)\frac{dY(t)}{dt}+\frac{Y(t)}{k(t,Y,X)}=X(t) \tag{3}$$

where: $Y(t)$, runoff; $\tau(t, Y, X)$, concentration time (Relaxation coefficient); $k(t, Y, X)$, runoff coefficient.

Thus τ *y* k depends on time, runoff and precipitation: this fact makes equation (3) nonlinear because of the coefficients relationship with the runoff we are trying to predict in the moment $t + \Delta t$. Equation (3) can be solved applying Newton's iteration method and using the subsequent finite difference scheme with a defined tolerance level ε:

$$\frac{\tau^t}{\Delta t}(Y^{t+\Delta t}-Y^t)+\frac{Y^{t+\Delta t}}{k^{t+\Delta t}}=X^{t+\Delta t} \tag{4}$$

In the end, the coefficients (τ *y* k) have to be parameterized and identified applying the next equation that links τ with runoff (Y) roughness coefficient (n), terrain slope (So) and distance to basin outlet (l):

$$\tau=\frac{n}{Y^{2/3}}\frac{l}{So^{1/2}} \tag{5}$$

and solving the inverse problem that leads to the relationship:

$$k^{t+\Delta t}=(Y^t)^a\left(X^{t+\Delta t}-\frac{\tau}{\Delta t}(Y^{t+\Delta t}-Y^t)\right)^b 10^c \tag{6}$$

Here, a, b and c are the inverse problem coefficients that are identified through historical data where data is available (using multiple regression analysis) or by generalization principles (spatial interpolation), e.g. in ungauged zones.

INTRODUCING UNCERTAINTY

To introduce some metrics of uncertainty in (3), lets say that $-(1/k\tau) = C$ and $X/\tau = N$, then C represents the basin's properties and N is the external influence over the basin domain. Both C and N can be represented as a composition of structural and random components by $C=\overline{C}+\tilde{C}$ and $N=\overline{N}+\tilde{N}$ where $\tilde{C},\tilde{N}$ are white noise processes (or processes without memory) with noise intensities $G_{\tilde{C}},G_{\tilde{N}}$ (Pankratov, 1999), this way (3) assumes the form:

$$\frac{dY(t)}{dt}=(\overline{C}+\tilde{C})Y(t)+(\overline{N}+\tilde{N}) \tag{7}$$

Integrating (7) wet set:

$$Y(t+\Delta t) = Y(t) + \int_{t}^{t+\Delta t} (\overline{C} + \widetilde{C}) Y dt + \int_{t}^{t+\Delta t} (\overline{N} + \widetilde{N}) dt \tag{8}$$

Equation (8) is the generalized stochastic rainfall–runoff model (Kovalenko, 1993). From (8) we can deduce that $Y(t + \Delta T)$ depends just from the value of $Y(t)$ and taking in account that $\tilde{C}$ and $\tilde{N}$ are white noises we obtain that $P(x_n, t_n \mid x_1, t_1; \ldots; x_{n-1}, t_{n-1}) = P(x_n, t_n \mid x_{n-1}, t_{n-1})$. From this it follows that equation (8) fulfils all conditions to be a Markov process (it would not be if we do not use a deterministic kernel with the same structure of equation (3) or under another kind of noise). To explain a Markov process it is necessary to describe the evolution of conditional probabilistic density $P(x_n,t_n \mid x_{n-1}, t_{n-1})$ for which we can obtain the following partial differential equation (Rozanov, 1979; Kolmogorov, 1986; Kovalenko, 1993; Frish, 1998):

$$\frac{\partial P(Y,t)}{\partial t} = -\frac{\partial}{\partial Y}\left[A(Y,t)P(Y,t)\right] + 0.5\frac{\partial^2}{\partial Y^2}\left[B(Y,t)P(Y,t)\right] \tag{9}$$

In this equation P represents the probabilistic density and $A(Y,t)$ and $B(Y,t)$ are the drift and diffusivity coefficients. They can be given by:

$$A(Y,t) = -(\overline{C} - 0.5G_{\tilde{C}})Y - 0.5G_{\tilde{C}\tilde{N}} + \overline{N} \tag{10}$$

$$B(Y,t) = G_{\tilde{C}}Y^2 - 2G_{\tilde{C}\tilde{N}}Y + G_{\tilde{N}} \tag{11}$$

where: $G_{\tilde{C}\tilde{N}}$, cross intensities for basin parameters and external influence.

Equation (9) provides a full description of a Markov process by means of $P(x_n,t_n \mid x_{n-1}, t_{n-1})$ evolution in time. Equation (9) is homogenous (there are no perturbation entries to the FPK equation) in partial derivatives, but due to equations (10) and (11), relationships that provide a bridge between the deterministic kernel (3) and the equation for description of a random process (9), we do not reject the cause–effect principle, precipitation as a system generator factor is included in the drift coefficient relationship and also the noise intensity is included in the formulae that determine diffusivity coefficient behaviour. This ensures an *a posteriori* PDC forecast through FPK equation solution.

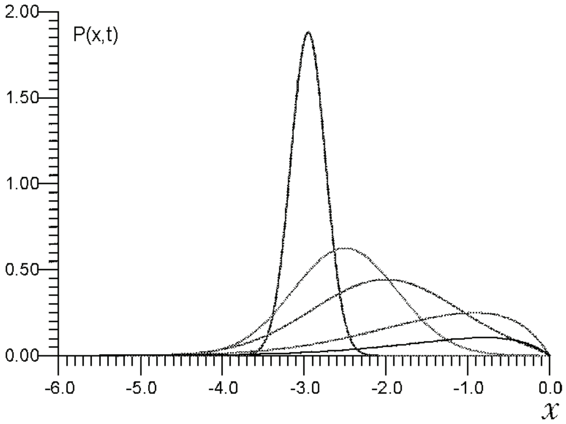
DYNAMIC SOLUTION OF FPK EQUATION

The solution for equation (9) may be either analytical or numerical. Analytical solutions can be found for a few problems with very strong restrictions. The numerical solution is able to solve a wide spectrum of general problems. As Dominguez (2002) showed, equation (9) can be solved by applying the next numerical explicit bidirectional scheme (NEBS):

$$P_j^{i+1} = -\left[krd\,\frac{A_{j+1}^i P_{j+1}^i - A_j^i P_j^i}{\Delta Y} + kld\,\frac{A_j^i P_j^i - A_{j-1}^i P_{j-1}^i}{\Delta Y}\right]\Delta t + 0.5\left[\frac{B_{j+1}^i P_{j+1}^i - 2B_j^i P_j^i + B_{j-1}^i P_{j-1}^i}{\Delta Y^2}\right]\Delta t + P_j^i \tag{12}$$

where *krd* and *kld* are direction coefficients (they enable two directional drift). The scheme (9) has to satisfy the following stability condition: $\frac{\Delta t \left|\max[B(Q,t)]\right|}{\Delta Q^2} \le \frac{1}{2}$.

To properly complete the formulation of a Cauchy-Dirichlet problem we must provide an initial probability density curve PDC at time $t = t_0$ and also define the boundary conditions according to the essence of the task. There are two kinds of boundary conditions that are widely applied (Pankratov, 1999), those are absorbing and reflecting boundaries (Illustration 1). Nevertheless, readers may find more boundary types in Gardiner (1985).



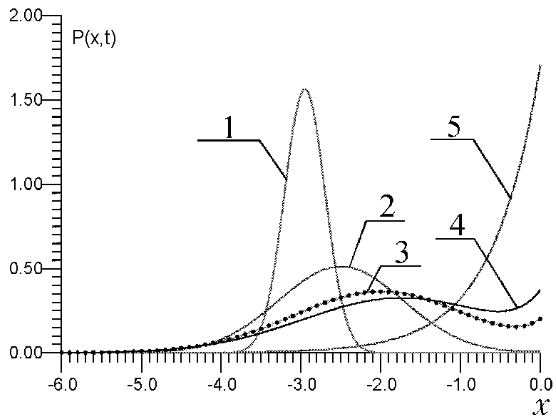


Fig. 1 Absorbing and reflecting boundary conditions for FPK equation.

QUASI—STATIONARY SOLUTION OF FPK EQUATION

In the case of climate change evaluation, we do not need to forecast all the *a posteriori* PDC. For the climate change scenarios generation it is enough to be able to forecast the three first statistical moments for the *a posteriori* PDC. The general tendency is to provide the new climatological scenarios without the continuous simulation of all the warming process (IDEAM, 2001). This means for hydrological scenarios building we have just an initial climatology and one forecast climatology. This forecasted climatology represents the external influence for which we have to build the hydrological response (Dominguez, 2001). Therefore the full dynamic FPK solution does not make sense; instead it is better to suppose[1] (IDEAM, 2001; Dominguez, 2002) that $\partial P(Q,t)/\partial t = 0$, then FPQ equation can be simplified and it leads to:

$$\frac{dP}{dY} = \frac{Y-a}{b_0 + b_1 Y + b_2 Y^2} P \tag{13}$$

From the FPQ equation we have obtained the classical formulation for the Pearson equation where:

[1]This assumption is supported by the fact that climate scenarios are given in a jump form from a start position to a forecasted scenario without transitive process.

$$a = \frac{G_{\tilde{C}\tilde{N}} + 2\overline{N}}{2\overline{C} + G_{\tilde{C}}} \quad b_0 = -\frac{G_{\tilde{N}}}{2\overline{C} + G_{\tilde{C}}} \quad b_1 = \frac{G_{\tilde{C}\tilde{N}}}{2\overline{C} + G_{\tilde{C}}} \quad b_2 = -\frac{G_{\tilde{C}}}{2\overline{C} + G_{\tilde{C}}} \tag{14}$$

Pearson coefficients through the FPQ equation have acquired specific physical sense, adopting a relationship with noise intensities (at the same time noise intensities have a very close relationship with human activities in the basins). It is possible to deduct the formulation to simulate statistical moments' evolution (Kovalenko, 1993) from equation (9). Using a well known procedure from equation (9) we obtain:

$$\frac{dm^n}{dt} = nM[A(Y,t)Y^{n-1}] + 0.5n(n-1)M[B(Y,t)Y^{n-2}] \tag{15}$$

For the first three statistical moments (n = 1 … 3) and remembering that there are no transitions $dm^n / dt = 0$, equation (15) becomes an algebraic equation system as follows:

$$nb_0\alpha_{n-1} + \left[(n+1)b_1 - a\right]\alpha_n + \left[(n + 2b_2 + 1)\right]\alpha_{n+1} = 0 \tag{16}$$

The equation system has to be parameterized using historical data which allows forecasting the three first statistical moments for the new climatic conditions (Dominguez, 2001).

MODELLING DOMAIN OR WHERE TO FORECAST

The above presented mathematical apparatus has to be developed for the nationwide domain of the Colombian Republic. Both the system dynamic and quasi–stationary solutions are solved in a 10 × 10 km grid providing the PDC simulation for all 11 140 nodes that cover the Colombian territory.

At the time frames, simulation is supported at two levels, monthly and long-term (more than 10 years). The monthly level can be used to support the hydropower generation and the long-term forecast is oriented to support the formulation of long planning policies at governmental level (e.g. for adaptation to climate changes).

INPUT DATA AND SPATIAL DISTRIBUTION OF PARAMETERS IN UNGAUGED BASINS

This research is supported by data provided by the hydrometeorological network that IDEAM operates (about 2000 meteorological and 800 hydrological stations). This network is managed as an integrated network and includes nodes for general and specific hydrometeorological characterization. Nevertheless, there are some regions in Colombia with a very poor network coverage (e.g. Caribbean, Amazonian and eastern flat lands regions). In these regions, data spatialization provides a very poor resolution for model inputs, and in some nodes we can find contradictions (e.g. more runoff than rainfall). There are no automatic interpolations schemes to enable the generation of hydrometeorological input layouts without incongruence; in some situations the interpolation scheme generates useless information. To fulfill all nodes (including nodes in ungauged basins) input of data in this research was implemented by a forcing data scheme supported by the FPK modelling system. In this scheme the first step consists of a preliminary interpolation of hydrometeorological data using the Gandin or Kriging method with a spherical model for the structural function. Precipitation and runoff layouts obtained by this method have to contain incongruence. However, with those layouts the FPK model is parameterized, solving the inverse problem. Obtained parameters are checked by experts who have to provide threshold values for each parameter. Filtering the obtained parameters with threshold values we get a time series for parameters that contains some void (rejected) values that have to be refilled with the FPK modelling system parameterized with no rejected data. The goodness of fit is tested in this step. If it does not reach the required level, the process is repeated from the preliminary interpolation changing the structural function model (Fig. 2). There will always be some nodes where the required precision will not be reached.

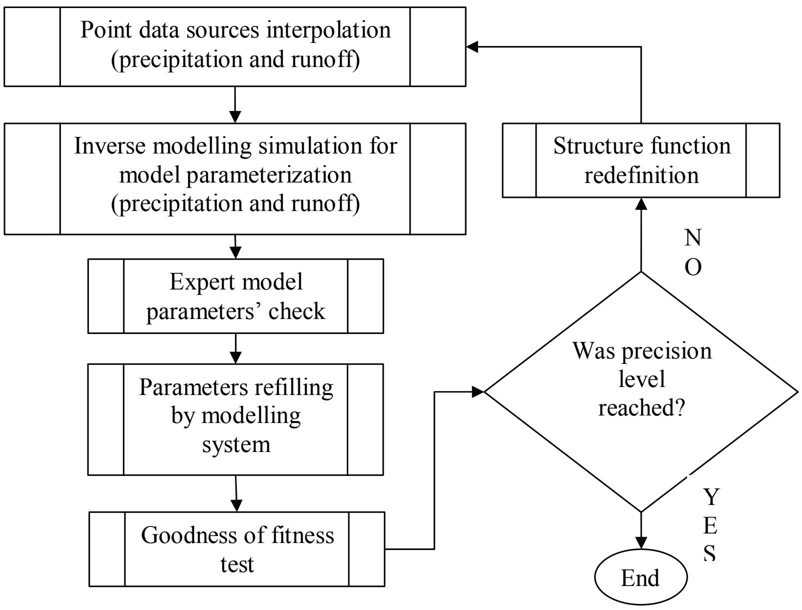


Fig. 2 Forcing data scheme with modelling support (FDSMS).

The application of FDSMS allows IDEAM to build hydrological scenarios with PDC simulation at any node in the modelling domain. The refilling procedure provides data over the region of influence for ungauged basins. The precision level was reached according to the results of Bayesian Information Score (BIS (NOAA-NWS forecast performance metric)) and S/σ (Russian Hydrometeorological Centre forecast performance metric) performance metrics (Dominguez, 2002).

CONCLUSIONS

The technologies presented above (FPK equation and FDSMS) provide added value to hydrometeorological products because of their user oriented design. The FPK equation is a unique technique able to forecast *a posteriori* PDC for SDMP and FDSMS is a objective way to generate hydrological data in a coarse grid (10 × 10 km). The modelling support plus experts parameter checks improve the hydrological data estimation for ungauged basins in the sense of exact replication of model exercise.

The results for quasi stationary solutions for the FPK equation are published in IDEAM (2001). Actually, the build scenario represents a forecast for the *a posteriori* PDC that can occur in the case of an increase of CO_2 concentration on the Earth's atmosphere. Obtained results are presented in terms of the first statistical moments. Summarizing, global warming (CO_2 duplication scenario) in Colombia could appear as several hydrological regime changes in the Amazon region and eastern flat lands. In those regions the forecasted hydrological scenario shows that hydrological regimes can become a bimodal regime due to the increase of precipitation in the forecasted precipitation scenarios. For some regions of the Colombian Republic the possible changes determine a runoff average increase with a temporal variability reduction, and for another region a runoff average reduction with a temporal variability increase is expected. In both cases changes in the PDC asymmetry are not expected. In IDEAM (2001) some conclusions are reported in the sense of the flood frequency increase, but with a magnitude reduction of flood discharges values.

The results obtained are very important and can be used directly to study the sensibility of productive sectors to hydrological regime changes; it can be done by introducing the above-mentioned characteristic in the production function for each sector (Kovalenko, 1993). Those functions are sensitive, not only to capital inversion, and to hydrological inputs, but also the way existing industries and the agro sector are adapted to the present hydrology because of their hydrology-oriented design or their optimization along the time of their development. This allows the formulation of the hypothesis of productive sectors' vulnerability to changes in hydrological regime as presented in IDEAM (2001).

The solution of the dynamic FPK equation at the monthly level provides a way to formulate operation rules with a well balanced cost–benefits relationship because of the short base *a posteriori* PDC. This optimization is reached using the production function of a hydropower system coupled with the stochastic water balance of the reservoir as suggested by Dominguez (2002). The enhanced hydrological forecast of *a posteriori* PDC enables managers of water system devices to improve risk assessment procedures through inverse problem solution for SDMP (What if? analysis). This technique allows the formulation of operation rules, together with ecosystem conservation directives. All stated advantages provide a great added-value to the hydrological forecast. The added value can be maximized, increasing the time resolution of the FPK equation to the daily level. This kind of improvement requires the use of remote sensing technologies (satellite imagery, and radar defined rainfall fields). At the daily level the FDSMS acquires more and more importance and has to be supported by a computer-based expert system to replace the human expertise in the parameters' check step, enabling the simulation system to operate in real time.

At the same time, with all the advantages of enhanced probabilistic hydrological forecast, problems with the noises intensity identification still remain. Actually results were produced applying parameter optimization techniques that avoided physical interpretation of mentioned intensities: those intensities have to be obtained from external conceptual domain to allow the forecast of *a posteriori* PDC without the application of no-frozen noise intensities hypothesis.

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